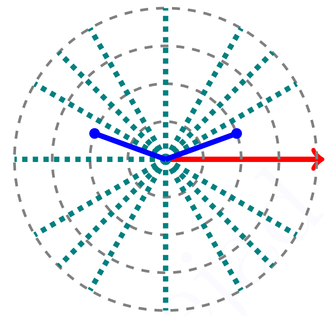
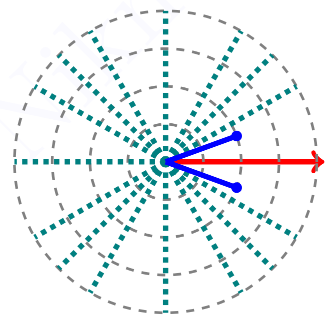
8.4: Polar Coordinates: Graphs

- Three rules to find the symmetries of a polar graph:

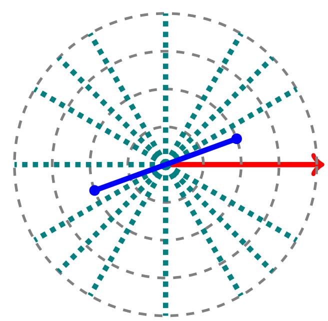
  1. If replacing $(r,\theta) \to (-r,-\theta)$ or $(r,\theta) \to (r,\pi-\theta)$ does not change the equation, then symmetry is about $\theta = \dfrac{\pi}{2}$ (y-axis).



  2. If either replacing $(r,\theta) \to (r,-\theta)$ or $(r,\theta) \to (-r,\pi-\theta)$ does not change the equation, then the symmetry is about $\theta = 0$ (or polar axis or x-axis).



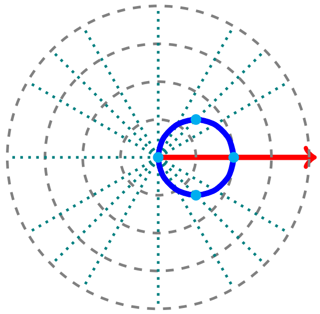
  3. If replacing $(r,\theta) \to (-r,\theta)$ or $(r,\theta) \to (r,\theta+\pi)$ does not change the equation, then the symmetry is about the pole (origin).



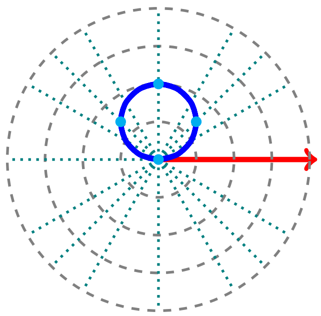
- How to graph
  - Find the symmetries of the graph. Use identities $\cos(-\theta) = \cos(\theta)$, $\sin(-\theta) = -\sin(\theta)$, $\cos(\pi-\theta) = -\cos(\theta)$ and $\sin(\pi-\theta) = \sin(\theta)$. Be aware that some symmetries don't reveal themselves until you graph. The only way to check for all symmetries is to check all three rules with $2k\pi$ added to angles for each integer k.
  - Find points where the max and the min of the trigonometric functions happen.
  - Find values of θ where $r = 0$.
  - Find extra points. Make a table.
  - Graph.

- Equations of the Form $r = a + b\sin(\theta)$ or $r = a + b\cos(\theta)$

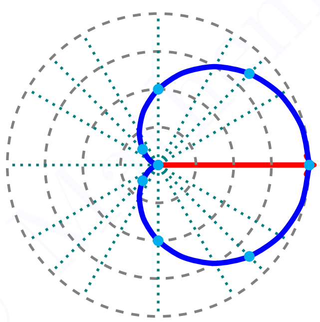
1. Circle $r = a\cos(\theta)$.



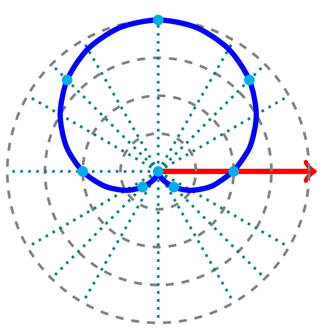
2. Circle $r = a\sin(\theta)$.



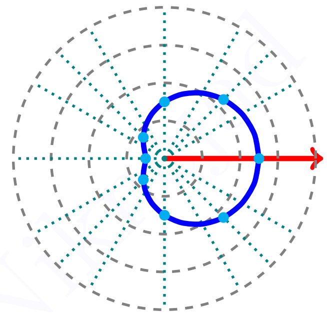
3. Cardioid $r = a + a\cos(\theta)$. (General form $r = a \pm a\cos(\theta)$ or $r = a \pm a\sin(\theta)$ for $a > 0$.)



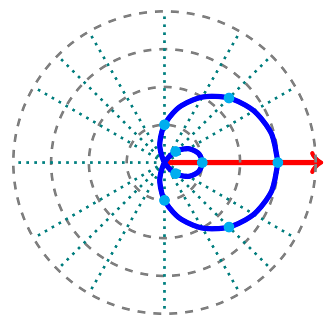
4. Cardioid $r = a + a\sin(\theta)$.



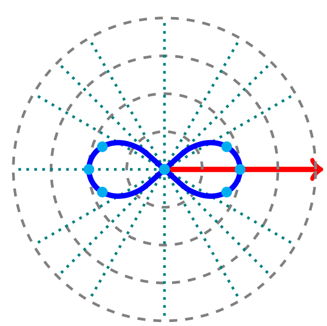
5. One Loop Limaçon $r = a + b\cos(\theta)$, $a > b > 0$. (General form is $r = a + b\cos(\theta)$ or $r = a + b\sin(\theta)$ where $a > b > 0$.) If also $a < 2b$, the indent will appear, a.k.a. dimpled Limaçon.



6. Inner Loop Limaçon $r = a + b\cos(\theta)$, $b > a > 0$. (General form is $r = a + b\cos(\theta)$ or $r = a + b\sin(\theta)$ where $b > a > 0$.)

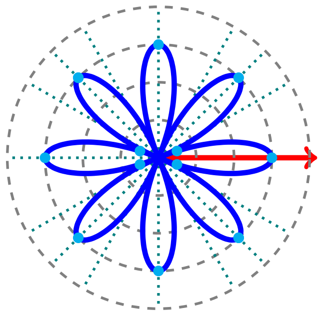


7. Lemniscate $r^2 = b\cos(2\theta)$, $b > a > 0$. (General form is $r^2 = \pm b\cos(2\theta)$ or $r^2 = \pm b\sin(2\theta)$.) This is tricky on the calculator since not all of the points will show up in the process of square rooting.

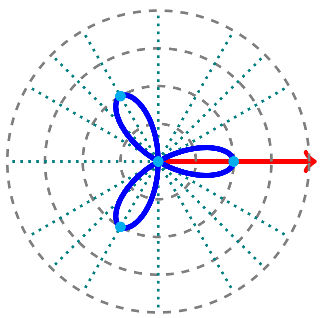


- Other Graphs:

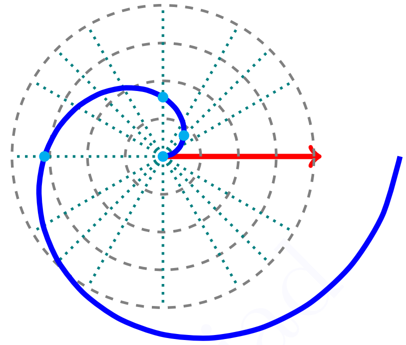
1. Rose Curve $r = b\cos(4\theta)$. (General form is $r = b\cos(n\theta)$ or $r = b\sin(n\theta)$. If n is even , $2n$ petal will show. If n is odd, n pedal will show.)



2. Rose Curve $r = b\cos(3\theta)$.



3. Archimedes' Spiral $r = \theta$ for $\theta > 0$.



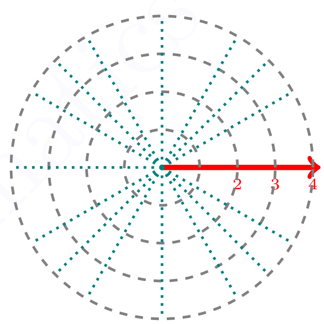
To view a computer generated graph of Rose Curve, go to https://www.geogebra.org/graphing/nb8j4ygn. Enter n and click play on slider a.

A Quick Tip: When graphing, it pays off to remember the approximate shape of the graph and choose your points to plot accordingly.

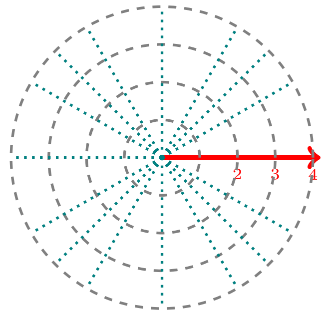
1. Find the symmetries of the following polar equations using the three rules.

   (a) $r = 2 - 2\cos(\theta)$ (b) $r^2 = \sin(\theta)$ (c) $r = \sin(\theta)$

2. Graph $r = 1 + 2\sin(\theta)$ on interval $[0, 2\pi]$.



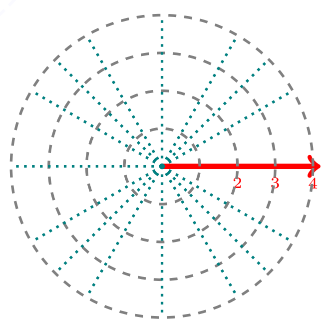
3. Graph $r = 3\sin(2\theta)$ on the interval $[0, 2\pi]$. Label each pedal in the order they appear in the domain.



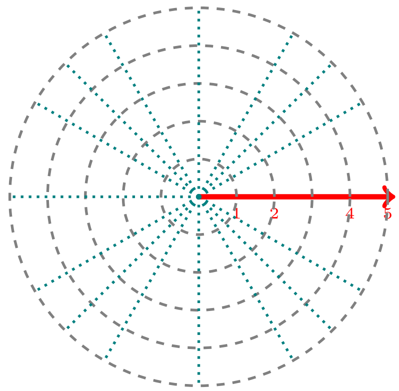
4. (a) Graph $r = 3.5\ \sin(5\theta)$ on the interval $[0, \pi]$.

   (b) Mark the piece of the graph restricted to domain $\left[\frac{\pi}{5}, \frac{2\pi}{5}\right]$.

   (c) What happens if we sketch over the domain $[0, 2\pi]$?



5. Graph $r = 0.5\ \theta$ for $0 \le \theta \le 3\pi$.



Example Video:

https://mediahub.ku.edu/media/t/1_sxz2xve3